This diagnostic test consists of two parts, **Fundamentals** and **Problem Solving**. If your student can solve nearly all of the **Fundamentals** problems and at least half of the **Problem Solving** problems, then the student is ready for the Art of Problem Solving Online Class **Math 7: Introduction to Algebra (Part 1)**.

If the student cannot solve more than 80% of the **Fundamentals** section, then the student should consider our **Math 6: Prealgebra (Part 1)** course. If the student solves nearly all of the **Fundamentals** but has a great deal of difficulty with the **Problem Solving** problems, then the student should consider our **Math 6: Prealgebra (Part 2)** course, which will help the student develop the maturity and problem-solving skills needed for our **Math 7: Introduction to Algebra (Part 1)** course.

We recommend using the following process in administering this diagnostic:

> Step 1: The student should attempt all of the questions below without a calculator and without any help. There is no time limit.
> Step 2: Check the student's responses using the answer key at the end of this document.
> Step 3: The student should be given a second chance on the problems that they answered incorrectly.

Fundamentals

1. **Working with Variables**.
   (a) Expand the product $9(3x+7)$.
   (b) Simplify $7a-5b+3(6a+b)$.
   (c) Simplify $a-(-7a-3)$.
   (d) Simplify $3(5-2r)-2(-3r+1)$.
2. **Fractions, Decimals, and Basic Percents**.
   (a) Convert 0.6144 to a fraction in simplest form.
   (b) Convert $17/40$ to a decimal.
   (c) 17 is what percent of 20?
   (d) What is $\frac{9}{5}-\frac{4}{3}$?
   (e) What is $\frac{3/6}{2/5}$ in simplest form?
3. **Linear Equations**. Solve each of the following equations:
   (a) $3r-4=16-7r$ (b) $\dfrac{2x-3}{5}=\dfrac{4-3x}{7}$ (c) $2-\dfrac{t}{4}=3\left(5-\dfrac{t}{6}\right)$
4. **Exponent Laws**. Express each of the following as a power of 2:

(a) $2^7 \cdot 2^6$ (b) $\frac{2^{13}}{2^5}$ (c) $(2^4)^3$ (d) $2^3 \cdot \left(\frac{2^7}{2^2}\right)^3$

5. **Ratio and Rates**.
   (a) The ratio of dogs to cats at an animal shelter is 4 to 5. If the total number of animals (dogs and cats) at the shelter is 108, then how many dogs are at the shelter?
   (b) The ratio of teachers to students in a particular school is 1 to 11. The ratio of 7th grade students to the total number of students is 4 to 9. If there are 396 students in 7th grade, then how many teachers are there?
   (c) A train is traveling 1 mile every 75 seconds. If the train continues at this rate, then how far will it travel in two hours?
6. **Square Roots**. Simplify each of the following as much as possible:

(a) $\sqrt{81}$ (b) $\sqrt{144}$ (c) $\sqrt{1\frac{7}{9}}$ (d) $\frac{\sqrt{540}}{\sqrt{3}}$

Problem Solving

7. What is the value of the sum $5 + 10 + 15 + \cdots + 95 + 100$?
8. Two-fifths of the students at Central Middle School play soccer. One-third of the non-soccer-players play the piano and one-quarter of the soccer-players play the piano. What fraction of the students in Central Middle School play the piano?
9. Kayla adds the same number to both the numerator and denominator of the fraction $\frac{1}{10}$. Her resulting fraction equals $\frac{2}{3}$. What number did she add to both the numerator and denominator of her original fraction?
10. Five workers together can build a road in 20 days. Suppose every worker works at the same rate. Three workers work on the road for 10 days before eleven more workers join them. How much longer will it take the fourteen workers to finish the road?
11. In rectangle $ABCD$, point X is the midpoint of $\overline{AD}$ and Y is the midpoint of $\overline{CD}$. What fraction of the area of the rectangle is enclosed by $\triangle AXY$?
12. At Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?

Don't look at the next page until you've attempted all the problems!

The answers are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. (a) $27x + 63$
   (b) $25a - 2b$
   (c) $8a + 3$
   (d) 13
2. (a) $384/625$
   (b) 0.425
   (c) 85%
   (d) $7/15$
   (e) $5/4$
3. (a) $r = 2$
   (b) $x = \dfrac{41}{29}$
   (c) $t = 52$
4. (a) 2^{13}
   (b) 2^{8}
   (c) 2^{12}
   (d) 2^{18}
5. (a) 48 dogs
   (b) 81 teachers
   (c) 96 miles
6. (a) 9
   (b) 12
   (c) $\frac{4}{3}$ or $1\frac{1}{3}$
   (d) $6\sqrt{5}$
7. 1050
8. $\dfrac{3}{10}$
9. 17
10. 5 days
11. $\frac{1}{8}$

12. 40