Do You Know? Math 7: Introduction to Algebra (Part 2)

If you can solve nearly all of the following problems with little difficulty, then the Art of Problem Solving class **Math 7: Introduction to Algebra (Part 2)** would only serve as a review for you.

1. The expression $x^5 + y^5$ can be written as the product of $x + y$ and another factor. Find that other factor.
2. If $x = \frac{1 - i\sqrt{3}}{2}$, then what is $\frac{1}{x^2 - x}$?
3. Find all values of z such that $z^4 - 4z^2 + 3 = 0$.
4. Find the radius and the center of the circle that is the graph of the equation $4x^2 + 4y^2 + 4x - 16y = 7$.
5. If $f(x) = ax^4 - bx^2 + x + 5$ and $f(-3) = 2$, then what is $f(3)$?
6. For how many positive integers b is $\log_b(729)$ a positive integer?
7. For what real values of x is $(1 - |x|)(1 + x)$ positive?
8. A rubber ball is dropped from a 100 ft tall building. Each time it bounces, it rises to three-quarters its previous height. So, after its first bounce it rises to 75 ft, and after its second bounce it rises to 3/4 of 75 ft, and so on forever. What is the total distance the ball travels?
9. Find all solutions to the equation $\sqrt[3]{x^3 - x^2 - 10} = x - 1$.

Don't look at the next page until you've attempted all the problems!

The answers are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. $x^4 - x^3y + x^2y^2 - xy^3 + y^4$
2. -1
3. $\sqrt{3}$, 1, -1, and $-\sqrt{3}$
4. The radius is $\sqrt{6}$ and the center is $\left(-\frac{1}{2}, 2\right)$.
5. 8
6. There are 4 such integers: 3, 9, 27, 729.
7. It is positive when $x < -1$ or $-1 < x < 1$.
8. 700 ft
9. $3, -3/2$