Are You Ready? Math 7: Introduction to Algebra (Part 2)

This diagnostic test consists of two parts, **Fundamentals** and **Problem Solving**. If you can solve nearly all of the **Fundamentals** problems and at least half of the **Problem Solving** problems, then you are probably ready for the Art of Problem Solving Online Class **Math 7: Introduction to Algebra (Part 2)**. Answers to these problems are included at the end of this document. Do not use a calculator.

Fundamentals

1. **Solving Linear Equations**. Solve each of the following:
   (a) $3x - 7 = 9 - x$
   (b) $\frac{7-2r}{3} = 4r$
2. **Proportion**.
   (a) If x and y are directly proportional and $x = 8$ when $y = 20$, then what is y when $x = 40$?
   (b) If x and y are inversely proportional and $x = 8$ when $y = 20$, then what is y when $x = 40$?
3. **Systems of Equations**.
   (a) Find all values of r and s such that $r - s = 5$ and $3r - 5s = 9$.
   (b) Find all values of p and q such that $3p + 7q = 1$ and $5p = 14q + 53$.
4. **Graphing Lines**.
   (a) What is the slope of the line through $(3, 4)$ and $(-1, 3)$?
   (b) What is the slope of the graph of $3x - 4y = 7$?
   (c) Find the slope-intercept form of the equation of the line through $(5, -2)$ and $(-1, 6)$.
5. **Introductory Quadratics**. Find all solutions to the following equations:
   (a) $4x^2 = 81$
   (b) $x^2 + 8x + 12 = 0$
   (c) $x^2 - 3x - 88 = 0$

Problem Solving

6. A box containing 3 oranges, 2 apples, and one banana weighs 15 units. Another box containing 5 oranges, 7 apples, and 2 bananas weighs 44 units. A third box containing 1 orange, 3 apples, and 5 bananas weighs 26 units. How much does each fruit weigh?
7. Find z if $\dfrac{3}{1-\frac{2}{z}} = 3z$.
8. At a certain time, Janice notices that her digital watch reads a minutes after two o'clock. Fifteen minutes later, it reads b minutes after three o'clock. She is amused to note that a is six times the value of b. What time was it when she looked at her watch for the second time?
9. Calvin and Susie are running for class president. Of the first 80% of the ballots that are counted, Susie receives 53% of the votes and Calvin receives 47%. At least what percentage of the remaining votes must Calvin receive to catch up to Susie in the election?
10. The golden ratio is the largest number g such that $\dfrac{g}{g+1} = \dfrac{1}{g}$. Find $g(g-1)$.
11. A line passing through the points $(2a+2, 3a^2)$ and $(3a+4, 5a^2)$ has slope $a+3$. Find all possible values of a.
12. Describe all values of x that satisfy $7 - 3x < x - 1 \leq 2x + 9$.

Don't look at the next page until you've attempted all the problems!

The answers are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. (a) $x = 4$, (b) $r = \frac{1}{2}$
2. (a) 100, (b) 4
3. (a) $(r, s) = (8, 3)$, (b) $(p, q) = (5, -2)$
4. (a) $\frac{1}{4}$, (b) $\frac{3}{4}$, (c) $y = -\frac{4}{3}x + \frac{14}{3}$
5. (a) $x = \frac{9}{2}$, $x = -\frac{9}{2}$; (b) $x = -2$, $x = -6$; (c) $x = 11$, $x = -8$
6. Oranges weigh 1 unit, apples weigh 5 units, and bananas weigh 2 units.
7. 3
8. 3:09
9. 62%
10. 1
11. -1, 6
12. $x > 2$