

Students enrolling in **Scholars Math 11: Calculus** should be very comfortable with algebra (including complex numbers and logarithms), geometry (including analytic geometry), and trigonometry. In particular, **students who do not have a solid algebra background should take our *Scholars Math 9: Intermediate Algebra* course rather than Scholars Math 11: Calculus.**

Students entering this course should be able to solve all of the following problems—**without using a calculator**—with little or no difficulty.

Algebra

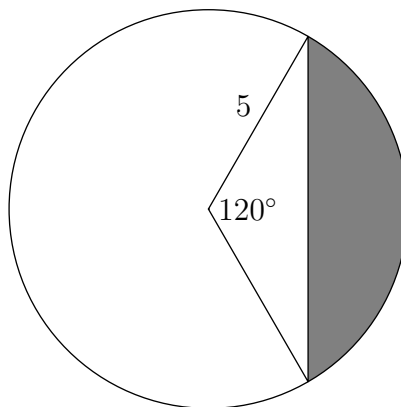
1. Expand $(x + 2y)^3$.
2. Find the sum and the product of the roots (real and complex) of $x^3 + 3x^2 + 7x - 11 = 0$.
3. Solve for x :

$$\frac{3}{x-2} + \frac{2}{x+2} = \frac{5}{x^2-4}.$$

4. If Richard can paint their living room in 4 hours, and Vanessa can paint the same living room in 5 hours, then how long will it take them to paint the living room working together?
5. Determine the sum of the infinite geometric series with first term 3 and common ratio $\frac{2}{5}$.
6. Compute $\log_9 27$.
7. Factor completely $x^6 - 1$ over the real numbers.

Geometry and Analytic Geometry

8. Find the area of the shaded region below (lying inside a circle of radius 5):



9. Find the equation of the line passing through the points $(2, 3)$ and $(5, -1)$.
10. Find the area of the region bordered by the lines $4x + 7y = 14$, $x = 1$, and $y = -2$.

11. Sketch the graph of the equation

$$x^2 + y^2 + 2x + 4y = 11.$$

Trigonometry

12. Evaluate the following quantities:

(a) $\sin\left(\frac{\pi}{6}\right)$

(b) $\cos\left(\frac{\pi}{2}\right)$

(c) $\tan\left(\frac{5\pi}{4}\right)$

13. Find all θ with $0 \leq \theta \leq 2\pi$ such that:

$$(\sin \theta + \cos \theta)^2 = \frac{3}{2}.$$

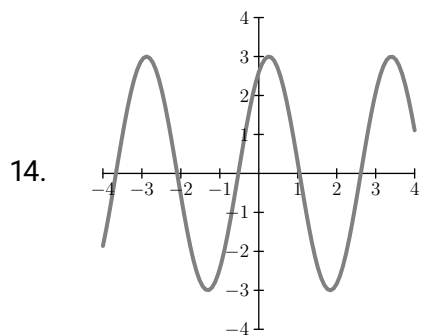
14. Sketch the graph of $y = 3 \sin(2x + 1)$.

15. Simplify $\left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)^9$.

Don't look at the next page until you've attempted all the problems!

The answers are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. $x^3 + 6x^2y + 12xy^2 + 8y^3$
2. Sum: -3 , Product: 11
3. $x = \frac{3}{5}$
4. $\frac{20}{9}$ hours
5. 5
6. $\frac{3}{2}$
7. $(x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1)$
8. $\frac{25}{3}\pi - \frac{25\sqrt{3}}{4}$
9. $(y - 3) = -\frac{4}{3}(x - 2)$ or $y = -\frac{4}{3}x + \frac{17}{3}$ or $4x + 3y = 17$ or equivalent
10. $\frac{72}{7}$
11. Circle with center $(-1, -2)$ and radius 4
12. (a) $\frac{1}{2}$ (b) 0 (c) 1
13. $\theta \in \left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \right\}$



The key features are passing through $(-\frac{1}{2}, 0)$, crossing the x -axis at periods of every π , and having an amplitude (height) of 3 .

15. $-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$