If you can solve nearly all of the following problems with little difficulty, then the Art of Problem Solving class **Scholars Math 9: Intermediate Algebra** would only serve as a review for you.

1. Solve for real and complex solutions to each of the following equations:
   (a) $7x^2 - 17x = -101$
   (b) $\sqrt{x-5} + \sqrt{x+15} = 10$
   (c) $\sqrt[3]{x^2-1} + \frac{20}{\sqrt[3]{x^2-1}} = 12$
   (d) $x^6 = 1$
2. The sum of the roots to a certain quadratic equation is 20. The product of the roots is 91. What are the roots of the quadratic?
3. Find integers x and y ($x > y$) that satisfy $x + y + xy = 223$ and $x^2y + xy^2 = 5460$.
4. Simplify this expression: $\sqrt[4]{161 - 72\sqrt{5}}$
5. Factor completely $6x^8 - 25x^7 - 31x^6 + 140x^5 - 6x^3 + 25x^2 + 31x - 140$.
6. If a and b are integers, what rational numbers x could satisfy the equation $6x^3 + ax^2 + bx = 35$?
7. Find integers a, b, and c such that the equation $x^4 + ax^3 + bx^2 + cx + 4 = 0$ has four distinct integer solutions.
8. For $x > 0$, find the minimum possible value of $4x + \frac{9}{x}$.
9. If $x + \frac{1}{x} = 5$, find the value of $x^5 + \frac{1}{x^5}$.
10. If $f(n)$ is a second degree polynomial such that $f(0) = 7$, $f(1) = 13$, and $f(2) = 23$, find $f(3)$.
11. What is the sum of the coefficients in the expansion of $(4x - 2y)^8$?
12. For how many of the first 500 natural numbers, n, will the equation $n = \lfloor 2x \rfloor + \lfloor 4x \rfloor + \lfloor 8x \rfloor + \lfloor 20x \rfloor$ have solutions?
13. Find (x, y, z) such that
    i. $x + y + z = 23$
    ii. $xy + yz + zx = 144$
    iii. $xyz = 252$
    iv. $x > y > z$
14. If $P(x)$ denotes a fifth degree polynomial such that $P(k) = \frac{k}{k+1}$ for $k =$0, 1, 2, 3, 4, and 5, determine $P(6)$.
15. Find all functions that satisfy the identity $f(x + 5y) + f(x - 5y) = 2x^2 + 50y^2$.
16. Prove that there is no polynomial $P(x)$ with integer coefficients such that $P(1) = 2$, $P(2) = 3$, and $P(3) = 1$.

Don't look at the next page until you've attempted all the problems!

The answers are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. (a) $\frac{17 \pm i\sqrt{2539}}{14}$
   (b) 21
   (c) ± 3, $\pm\sqrt{1001}$
   (d) ± 1, $\frac{1 \pm i\sqrt{3}}{2}$, $\frac{-1 \pm i\sqrt{3}}{2}$
2. 7 and 13
3. $x = 15$, $y = 13$
4. $\sqrt{5} - 2$
5. $(x-1)(x-4)(2x-5)(3x+7)(x^4+x^3+x^2+x+1)$ (Yes, there is a faster way than just plowing ahead with synthetic division.)
6. The 32 possible rational roots are all in the form $\pm\frac{m}{n}$ where m takes on each of the values 1, 5, 7, and 35, and where n takes on each of the values 1, 2, 3, and 6.
7. $a = 0$, $b = -5$, and $c = 0$
8. 12
9. 2525
10. 37
11. 256
12. 353
13. (14, 6, 3)
14. 1
15. $f(x) = x^2$
16. Consider the general polynomial $P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$. Notice that $P(r) - P(s) = a_1(r-s) + a_2(r^2 - s^2) + \ldots + a_n(r^n - s^n) = (r-s)Q(r,s)$ for some integer $Q(r,s)$. This means that $2 - 3 = P(1) - P(2) = -Q(1,2)$, $3 - 1 = P(2) - P(3) = -Q(2,3)$, and $1 - 2 = P(3) - P(1) = 2Q(3,1)$. This gives a non-integer $Q(3,1)$, which is a contradiction; thus, no such polynomial $P(x)$ exists.