

Students enrolling in **Scholars Math 9.1: Intermediate Counting & Probability** should be comfortable with geometric series, factoring and multiplying polynomials, and basic counting techniques. Examples of each are below.

Geometric Series

1. Evaluate the sum: $1 + 2 + 2^2 + 2^3 + \dots + 2^{10}$.
2. Assuming that $-1 < x < 1$, find a closed form expression for $1 - x + x^2 - x^3 + x^4 - x^5 + \dots$ by evaluating the sum as an infinite geometric series with common ratio $-x$.

Factoring and Multiplying Polynomials

3. Find the polynomial $f(x)$ such that $(x - 1)f(x) = x^6 - 1$.
4. Find the five terms with smallest degree of the product $(1 + x + x^2 + x^3 + x^4 + \dots)(1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots)$.

Counting Techniques

The following questions are from the "Do You Know Math 7.1: Introduction to Counting & Probability" quiz. If you cannot easily solve most of them, you should consider taking our **Math 7.1: Introduction to Counting & Probability** course before taking **Math 9.1: Intermediate Counting & Probability**.

5. How many multiples of 7 are between 83 and 229?
6. How many distinct arrangements are there of the letters in the word MATHEMATICS?
7. A coin is flipped, a 6-sided die numbered 1 through 6 is rolled, and a 10-sided die numbered 0 through 9 is rolled. What is the probability that the coin comes up heads and the sum of the numbers that show on the dice is 8?
8. Find the coefficient of x^3y^8 in the expansion of $(x - 2y^2)^7$.
9. Particle Man is at the origin in three-dimensional space. How many ways can Particle Man take a series of 12 unit-length steps, each step parallel to one of the coordinate axes, from the origin to $(3, 4, 5)$ without passing through the point $(2, 3, 2)$?
10. In poker, a hand is formed with 5 cards. The deck has 52 cards, separated into 4 suits. Each suit has 13 ranks which are the same in every suit. A full house occurs when a hand has 3 cards of one rank and 2 of another. How many different poker hands are full houses?
11. How many distinguishable ways can the faces of a regular hexagonal prism be painted 8 different colors (one color per face, no color used twice)?
12. There are $2n$ players in a chess tournament. The first round consists of pairing the players to participate in n matches with every player playing one match. In terms of n , how many ways can this pairing take place?

13. A playoff series between two teams proceeds one game at a time until one team has won 5 games. What is the probability that the series lasts 9 games if each team is equally likely to win each game?

Don't look at the next page until you've attempted all the problems!

The answers are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. 2047
2. $\frac{1}{1+x}$
3. $x^5 + x^4 + x^3 + x^2 + x + 1$
4. $1 + 3x + 6x^2 + 10x^3 + 15x^4 + \dots$
5. 21
6. 4989600
7. $1/20$
8. 560
9. 23520
10. 3744
11. 3360
12. $\frac{(2n)!}{2^n n!}$
13. $35/128$