AoPS Online
Art of Problem Solving

Do You Know? Scholars Math 7.1: Introduction to Counting & Probability

If you can solve nearly all of the following problems with little difficulty, then the class **Scholars Math 7.1: Introduction to Counting & Probability** would only serve as a review for you.

1. How many multiples of 7 are between 83 and 229?
2. How many distinct arrangements are there of the letters in the word MATHEMATICS?
3. A coin is flipped, a 6-sided die numbered 1 through 6 is rolled, and a 10-sided die numbered 0 through 9 is rolled. What is the probability that the coin comes up heads and the sum of the numbers that show on the dice is 8?
4. Find the coefficient of x^3y^8 in the expansion of $(x-2y^2)^7$.
5. Anna writes the first 1000 positive integers. She then circles the even ones with a green pen. Bob circles the multiples of three in red. Cindy circles the multiples of five in blue. How many numbers are circled exactly twice?
6. Particle Man is at the origin in three-dimensional space. How many ways can Particle Man take a series of 12 unit-length steps, each step parallel to one of the coordinate axes, from the origin to $(3,4,5)$ without passing through the point $(2,3,2)$?
7. In poker, a hand is formed with 5 cards. The deck has 52 cards, separated into 4 suits. Each suit has 13 ranks which are the same in every suit. A full house occurs when a hand has 3 cards of one rank and 2 of another. How many different poker hands are full houses?
8. How many distinguishable ways can the faces of a regular hexagonal prism be painted 8 different colors (one color per face, no color used twice)?
9. There are $2n$ players in a chess tournament. The first round consists of pairing the players to participate in n matches with every player playing one match. In terms of n, how many ways can this pairing take place?
10. Find two proofs that for every positive integer n, the following equality holds:

$$\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}+\ldots+(-1)^n\binom{n}{n}=0.$$

11. A playoff series between two teams proceeds one game at a time until one team has won 5 games. What is the probability that the series lasts 9 games if each team is equally likely to win each game?

Don't look at the next page until you've attempted all the problems!

The answers are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. 21
2. 4989600
3. $1/20$
4. 560
5. 233
6. 23520
7. 3744
8. 3360
9. $\frac{(2n)!}{2^n n!}$
10. One method is to let $x = y = 1$ in the binomial expansion of $(x - y)^n$. There are many others.
11. $35/128$