AoPS Online
Art of Problem Solving

## Do You Know? Scholars Math 10.1: Olympiad Geometry

If you are confident that you can solve nearly any geometry problem that might arise on the USAMO, and find the five problems below pretty easy, then **Scholars Math 10.1: Olympiad Geometry** would only serve as a review for you.

The final two problems in this list are examples of more challenging problems we will tackle in this class.

1. A point $D$ is placed on side $BC$ of triangle $ABC$. Circles are inscribed in $ABD$ and $ACD$. Their common exterior tangent (other than $BC$) meets $AD$ at $K$. Prove that the length of $AK$ is independent of $D$.
2. From vertex $A$ of triangle $ABC$, perpendiculars $AM$ and $AN$ are drawn to the bisectors of the exterior angles of the triangle at $B$ and $C$. Prove that $MN$ is equal to half the perimeter of $ABC$.
3. Given is cube $ABCDA'B'C'D'$ ($A$ connected to $A', B, C$, and likewise for the others). Points $M$ and $N$ are on $AA'$ and $BC'$ so that line $MN$ intersects line $B'D$. Find $\dfrac{BC'}{BN} - \dfrac{AM}{AA'}$.
4. $ABC$ is an acute-angled triangle. The incircle touches $BC$ at $K$. The altitude $AD$ has midpoint $M$. The line $KM$ meets the incircle again at $N$. Show that the circumcircle of $BCN$ is tangent to the incircle of $ABC$ at $N$.
5. Let $I$ be the incenter of triangle $ABC$. Prove that $(IA)(IB)(IC) = 4Rr^2$, where $R$ is the circumradius of $ABC$ and $r$ is the inradius of $ABC$.

**Don't look at the next page until you've attempted all the problems!**

Solution guides are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. See the ‘A Picture is Worth a Thousand Words’ section of the ‘How to Write a Solution’ article in the Articles subsection of the Resources page at www.artofproblemsolving.com.
2. See the ‘Follow the Lemmas’ section of the ‘How to Write a Solution’ article in the Articles subsection of the Resources page at www.artofproblemsolving.com.
3. $1$
4. Here’s a rough sketch of how to get to the solution. Let $P$ be where the extension of $NK$ meets the perpendicular bisector of $BC$. Then show that $(NK)(KP) = (BK)(KC)$, so that $P$ is on the circumcircle on $BCN$. Deduce that $N$, $I$, and $K$ are collinear, and hence the circles are tangent at $N$.
5. See the ‘Bookends’ section of the ‘How to Write a Solution’ article in the Articles subsection of the Resources page at www.artofproblemsolving.com.