This diagnostic test consists of two parts, Fundamentals and Problem Solving. If your student can solve nearly all of the Fundamentals problems and at least half of the Problem Solving problems, then the student is ready for the Art of Problem Solving Online Class **Scholars High School Physics 1: Mechanics**.

If the student cannot solve more than 80% of the Fundamentals section, the student should consider our **Math 7: Introduction to Algebra (Part 1)** course. If the student has a great deal of difficulty with the Problem Solving section, they should consider our **Scholars High School Physics: Foundations** course. These courses will teach the skills used on this test and help students develop their problem-solving skills.

In places, the numbers on this test are not selected to simplify the problem and are not whole numbers. This reflects how, in physics, we often work with numbers derived from experiment, not specially chosen examples. To help with the computations, students are allowed a calculator on this test, and will use calculators extensively in the class. Any calculator is fine.

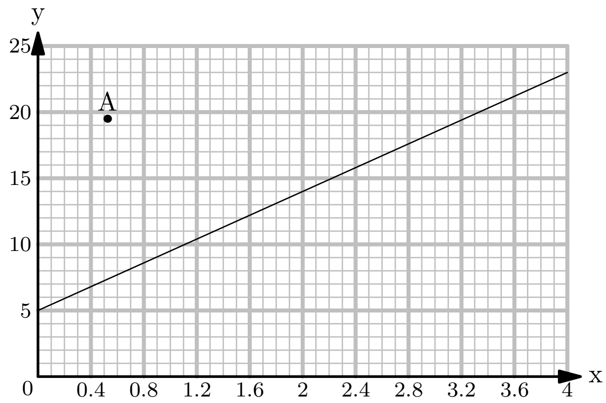
We recommend using the following process in administering this diagnostic:

> Step 1: The student should attempt all of the questions below without any help. **A calculator is allowed.** There is no time limit.
> Step 2: Check the student's responses using the answer key at the end of this document.
> Step 3: The student should be given a second chance on the problems that they answered incorrectly.

Fundamentals

1. **Computational Skills** Evaluate each of the following:
   (a) $\dfrac{(1.23)^2-(0.75)(0.15)}{0.83}$, rounded to the nearest hundredth.
   (b) 85.2% of 1200, rounded up to the next integer.
   (c) $4\left[\dfrac{2}{3}(10)\left(\dfrac{1}{4}\right)^3\right]$, expressed as a simplified fraction.
   (d) $\left[\sqrt{5}\left(\dfrac{8}{25}\right)^{\frac{1}{3}}\right]^6-\frac{2}{5}$, expressed as a simplified fraction.
2. **Algebra Skills**
   (a) Solve for x: $0.23x+5.7-1.11x=\frac{2.12x-0.4}{(0.25)^2}$. Report your answer to the nearest thousandth.
   (b) Solve for a and b: $3a=2-4b$ and $b=5a-11$.
   (c) If $v=\sqrt{\dfrac{2mg}{apc}}$, solve for p in terms of the other variables.

3. **Working with Graphs** Using the graph below, identify each of the following:

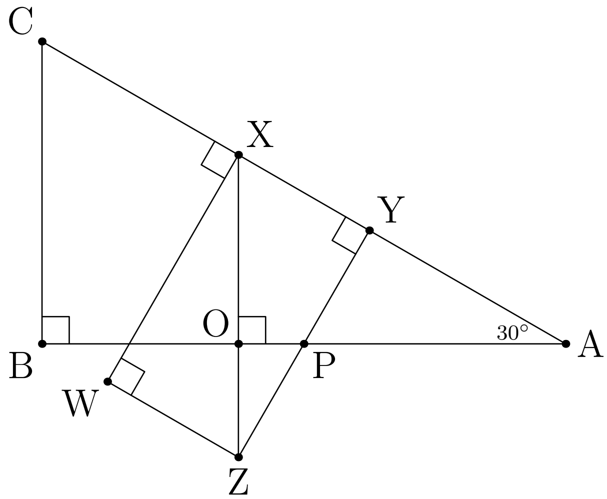


   (a) The coordinates of the point A.

   (b) The slope of the line.

   (c) The equation of the line.

4. **Angles**. In the diagram below, determine each of the following:



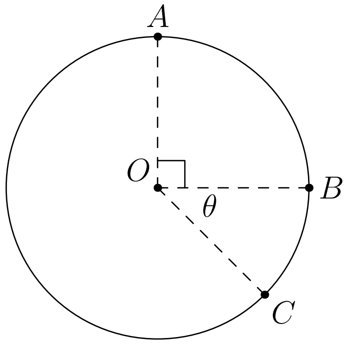
   (a) $\angle BCA$.

   (b) $\angle OPZ$.

   (c) $\angle WXZ$.

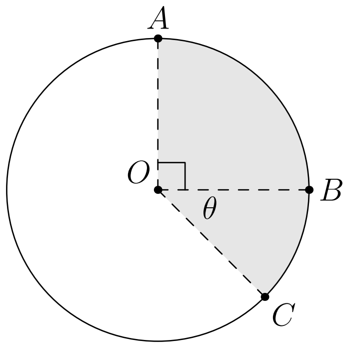
   (d) $\angle WZX$.

5. **Circles** The circle shown below has radius $r = 2$. Point O is the center of the circle and points A, B, and C lie on the circle. Compute the following:



(a) The length of arc $\overset{\frown}{AB}$.

(b) The area of sector AOC when $\theta = 45.0°$. (This sector is shaded below.)



(c) The area of a triangle formed by the following lines:

- Secant $\overleftrightarrow{AB}$.
- The line tangent to the circle at point A.
- The line tangent to the circle at point B.

Problem Solving

6. A team of scientists is considering four different hypotheses to explain a certain natural phenomenon. They label the hypotheses (a) through (d).

   The scientists then design four different experiments, $E1$ through $E4$.

   Each hypothesis has a prediction for what will happen in each experiment, so there are 16 total predictions. The predictions are labeled $F, G, H,$ etc.

   To organize all these predictions, the scientists build the chart shown below.

   If two rows of the chart have the same letter in a particular column, then those hypotheses make the same prediction for that experiment.

	E1	E2	E3	E4
Hypothesis a	F	G	I	K
Hypothesis b	F	G	J	L
Hypothesis c	F	G	J	K
Hypothesis d	F	H	I	L

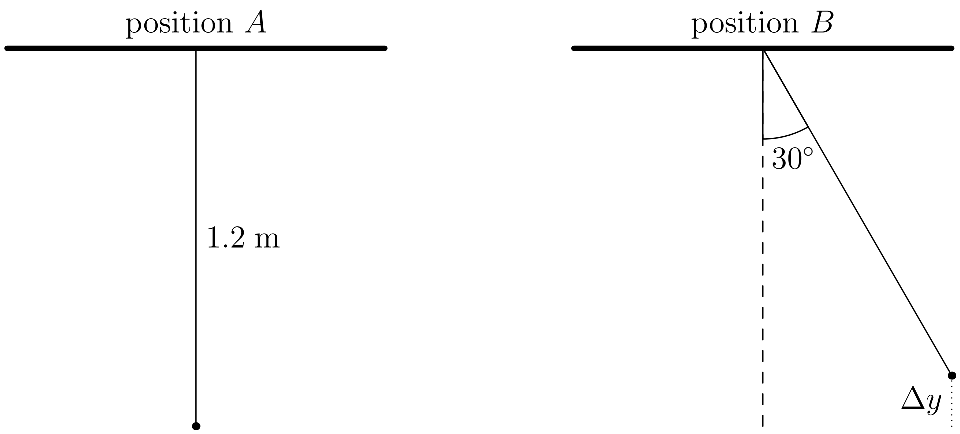
   Name exactly two experiments that the scientists could conduct in order to gain strong evidence that one particular hypothesis agrees with observations better than the other three. Also name one experiment which would not give strong evidence for any hypothesis over the others.

   You may assume that at least one of the hypotheses makes correct predictions for all four experiments.

7. A *pendulum* is a small weight at the end of a string which hangs from the ceiling.

   In position A, a pendulum hangs straight down. In position B, the pendulum has been raised so that it makes an angle of 30° with the vertical.

   If the length of the pendulum is $1.2\ \text{m}$, by how much, Δy, has the pendulum been raised in moving from position A to position B? Give your answer as a decimal, rounded to two decimal places.



8. In the SI unit system, the speed of light is set to $c = 299{,}792{,}458\ \mathrm{m \cdot s^{-1}}$ by definition. This is used to define the length of the meter; there is an independent definition of the second.

   Consider a (fictional) different unit system, which we will call ”round number units”. Round number units use the second as a base unit of time, defined the same way as in the SI system. They also use the same kilogram as SI. However, round number units have a unit called one ”step” as the base unit of length. The step is defined by setting the speed of light to

   $$c = 300{,}000{,}000\ \mathrm{step \cdot s^{-1}}.$$

   We would like to compare the energy units between SI and round number units.

   In SI, the unit of energy is the joule, defined as

   $$1\ \mathrm{J} = 1\ \mathrm{kg \cdot m^2 \cdot s^{-2}}.$$

   In round number units, the unit of energy is the energon, defined as

   $$1\ \mathrm{energon} = 1\ \mathrm{kg \cdot step^2 \cdot s^{-2}}.$$

   What is one joule of energy, measured in energons? Give your answer to at least five significant figures.

9. The Cretaceous-Paleogene extinction event was a mass extinction that occurred about 65 million years ago. It was responsible for the extinction of most dinosaurs and many other animals.

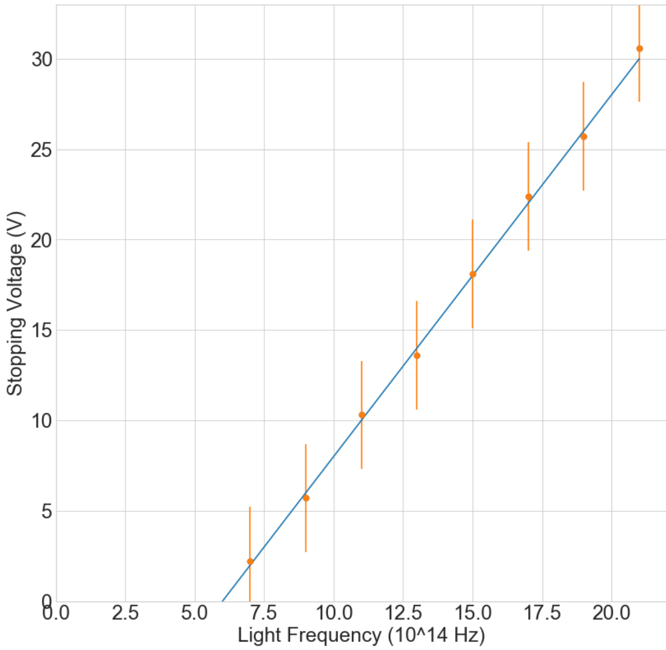
   Many scientists believe this extinction occurred when a large asteroid hit the Earth. Dust from the impact is supposed to have blocked a significant amount of sunlight, leading to cold temperatures that killed many animals.

   Scientists have estimated via computer simulations that about $\frac{1}{5}$ of the mass of an asteroid ends up as dust in the atmosphere. For the asteroid from the Cretaceous-Paleogene extinction, this dust eventually created a nearly-uniform layer of $0.02\ \mathrm{g/cm^2}$ over the surface of the Earth. The density of most asteroids is approximately $2\ \mathrm{g/cm^3}$. The radius of Earth is about $6400\ \mathrm{km}$. Estimate the radius of the asteroid to the nearest kilometer.

10. Some scientists conduct an experiment on the photoelectric effect. In their experiment, the scientists shine light of varying frequency on a metal sheet. The frequency of a light wave is measured in hertz (Hz). The frequency of light used is their independent variable.

The scientists measure something called the stopping voltage. This is the electric voltage needed to prevent electrons from jumping off the illuminated sheet of metal and onto another nearby sheet of metal. They measure the stopping voltage in volts (V).

The scientists conduct an uncertainty analysis, then plot their data along with error bars representing random error. The scientists also include a straight line representing the predictions of the theory of the photoelectric effect. Their results are shown below:



Based on the graph shown above, which of the following are likely true of the scientists' experiment? You may choose more than one.

(a) The scientists probably overestimated their uncertainty.

(b) The scientists probably underestimated their uncertainty.

(c) The scientists' uncertainty estimates are not a clear underestimate or overestimate.

(d) The data is a good fit to the theory.

(e) The data is a poor fit to the theory.

(f) The scientists' relative error was about the same for all data points.

(g) The scientists' relative error increased as the light frequency increased.

(h) The scientists' relative error decreased as the light frequency increased.

Don't look at the next page until you've attempted all the problems!

The answers are below.

1. (a) 1.69
   (b) 1023
   (c) $\dfrac{5}{12}$
   (d) $\dfrac{62}{5}$
2. (a) 0.348
   (b) $a = 2$; $b = -1$
   (c) $p = \dfrac{2mg}{v^2ac}$
3. (a) $(0.525, 19.5)$ (x coordinates in the range $0.51 \le x \le 0.54$ and y coordinates in the range $19.3 \le y \le 19.7$ are counted as correct.)
   (b) 4.5 or $\frac{9}{2}$ (Slope values in the range $4.4 \le m \le 4.6$ are counted as correct.)
   (c) $y = 4.5x + 5$ or $y = \frac{9}{2}x + 5$ (This is counted as correct if the slope in the previous problem was incorrect, but was used in place of the correct slope in this answer.)
4. (a) 60°
   (b) 60°
   (c) 30°
   (d) 60°
5. (a) π (Also acceptable: decimal form, e.g. 3.14.)
   (b) $\dfrac{3}{2}\pi$ (Also acceptable: decimal form, e.g. 4.71.)
   (c) 2
6. The two experiments the scientists should conduct are experiments $\boxed{\text{3 and 4}}$.

   When the scientists conducts experiment 3, either hypotheses (a) and (d) will have correct predictions, or hypotheses (b) and (c) will. So the next experiment needs to make different predictions for (a) and (d) and for (b) and (c). Only experiment 4 does that.

   The experiment that will not give any good evidence in favor of one hypothesis over the others is $\boxed{\text{experiment 1}}$. All the hypotheses make the same prediction for that experiment, so no hypothesis has made a better prediction than any other.
7. 0.16 m
8. $1 \text{ J} \approx \boxed{1.0014 \text{ energon}}$.

   The speed of light is the same quantity, no matter the units, so

$$c = 299{,}792{,}458 \text{ m} \cdot \text{s}^{-1} = 300{,}000{,}000 \text{ step} \cdot \text{s}^{-1}.$$

Solving for one meter, we find

$$1\ \text{m} = \frac{300{,}000{,}000}{299{,}792{,}458}\ \text{step}.$$

Next, we take the definition of the joule:

$$1\ \text{J} = 1\ \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2},$$

and substitute in the previous equation for the meter:

$$1\ \text{J} = 1\ \text{kg} \cdot \left(\frac{300{,}000{,}000}{299{,}792{,}458}\text{step}\right)^2 \cdot \text{s}^{-2}.$$

Using the definition of the energon, this is

$$1\ \text{J} = \left(\frac{300{,}000{,}000}{299{,}792{,}458}\right)^2\ \text{energon} \approx 1.0014\ \text{energon}.$$

The answer can include more digits and still be counted as correct. If the student found

$$1\ \text{J} = \left(\frac{300{,}000{,}000}{299{,}792{,}458}\right)^2\ \text{energon}$$

and did not use a calculator to find the decimal expansion, that can also be counted as correct.

9. The radius was about $\boxed{4\ \text{km}}$.

If the dust formed a uniform layer of $0.02\ \text{g/cm}^2$ using only 20% the mass of the asteroid, the entire asteroid would have left a layer five times as thick, or $0.1\ \text{g/cm}^2$.

We can find the mass of the asteroid by estimating the total mass in such a layer. The surface area of Earth is

$$A \approx 4\pi \cdot (6.4 \times 10^8\ \text{cm})^2 \approx 5.1 \times 10^{18}\ \text{cm}^2.$$

That makes the mass of the asteroid

$$m \approx 5.1 \times 10^{18}\ \text{cm}^2 \cdot 0.1\ \text{g/cm}^2 = 5.1 \times 10^{17}\ \text{g}.$$

Multiplying the volume of the asteroid to its density should give a second estimate of this mass, so the radius of the asteroid obeys

$$\frac{4}{3}\pi r^3 \cdot 2\ \text{g/cm}^3 \approx 5.1 \times 10^{17}\ \text{g}.$$

Solving for r, we find

$$r \approx 4 \times 10^5\ \text{cm} = 4\ \text{km}.$$

10. The correct choices are $\boxed{\text{(a), (d), and (h)}}$.

All the points on the plot are quite close to the line showing the theory's prediction. Meanwhile, the error bars are very large. Not only does the line representing the theoretical prediction go through every error bar, it goes through very close to the middle of every error bar.

The size of the error bar on the plot should represent the approximate random error in the experiment, meaning that the data points will vary up and down from the theoretical prediction by a distance similar to the size of the error bars, even if the theory is correct. Here, the points have much less variation from the theory than that, so the error bars are significantly $\boxed{\text{too large}}$.

Since all the data points are close to the line and there is no clear trend to the small variation from the line, the data is $\boxed{\text{a good fit}}$ for the theory.

All the error bars are about the same size. These error bars represent the absolute error. The relative error is $\dfrac{\text{absolute error}}{\text{stopping voltage}}$. As we increase the light frequency, we also increase the stopping voltage, so higher stopping voltages and higher frequencies lead to $\boxed{\text{lower relative error.}}$