

This diagnostic test consists of two parts: **Fundamentals** and **Problem Solving**. If your student can solve nearly all of the **Fundamentals** problems and at least half of the **Problem Solving** problems, then the Art of Problem Solving Online Class **Math 6: Prealgebra (Part 2)** would only serve as a review for your student.

We recommend using the following process to assess whether or not your student has already mastered the contents of this course:

Step 1: The student should attempt all of the questions below without a calculator and without any help. There is no time limit.

Step 2: Check the student's responses using the answer key at the end of this document.

Step 3: The student should be given a second chance on the problems that they answered incorrectly.

## Fundamentals

1. **Square Roots.** Simplify each of the following:

(a)  $\sqrt{81}$

(c)  $\sqrt{1\frac{7}{9}}$

(e)  $\sqrt{75} + \sqrt{27} - \sqrt{192}$

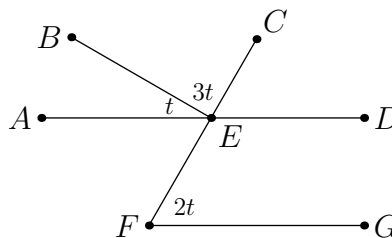
(b)  $\sqrt{144}$

(d)  $\frac{\sqrt{120}}{\sqrt{3}}$

(f)  $3\sqrt{18} + 7\sqrt{8}$

2. **Angles.**

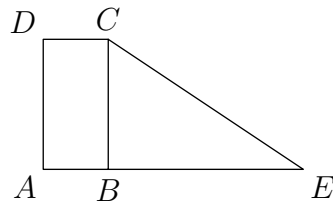
- (a) In the diagram below,  $\overline{AD}$  and  $\overline{CF}$  intersect at point  $E$ , and  $\overline{AD} \parallel \overline{FG}$ . We have  $\angle CEB = 3\angle AEB$  and  $\angle EFG = 2\angle AEB$ , as shown. Find the measure of  $\angle CED$  in degrees.



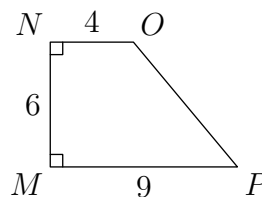
- (b) What is the measure of each interior angle of a regular polygon with 9 sides?

3. **Area.**

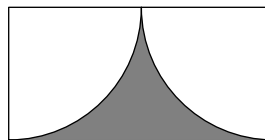
- (a) In the diagram below, rectangle  $ABCD$  and right triangle  $BCE$  have the same area. Find the ratio  $AB/AE$ .



- (b) Find the area of  $MNOP$  in the diagram below.



- (c) Two quarter-circles are drawn inside a rectangle as shown below. The two quarter-circles meet at a point on a side of the rectangle. If the radius of each quarter-circle is 6 inches, then what is the area of the shaded region?



#### 4. Pythagorean Theorem.

- (a) Bill walks  $\frac{1}{4}$  mile south, then  $\frac{3}{4}$  mile east, and finally another  $\frac{3}{4}$  mile south. How many miles is he, in a direct line, from his starting point?
- (b) The bases of a 39-foot pole and a 15-foot pole are 45 feet apart, and both poles are perpendicular to the ground. The ground is flat between the two poles. What is the length of the shortest rope that can be used to connect the tops of the two poles?
- (c) The base of an isosceles triangle is 24 and the area of the triangle is 60. What is the length of one of the equal sides?

#### 5. Quadrilaterals.

- (a) If  $EFGH$  is a parallelogram and  $\angle E = 41^\circ$ , then find the other angles of the parallelogram.
- (b) One base of a trapezoid has length 8 inches and the height of the trapezoid is 4 inches. If the trapezoid's area is 80 square inches, then what is the length of the other base of the trapezoid?
- (c) The lengths of the diagonals of a rhombus are 10 inches and 24 inches. What are the perimeter and the area of the rhombus?

**6. Basic Statistics.**

- (a) Compute the average (mean), median, and mode of the following numbers:

83, 84, 100, 91, 92, 96, 84.

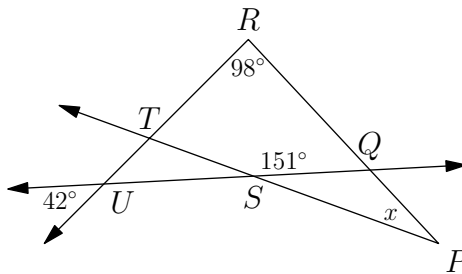
- (b) Suppose you average 82 on your first 7 tests in a class. What must you score on the eighth test to raise your average to 84?

**7. Counting and Probability Fundamentals.**

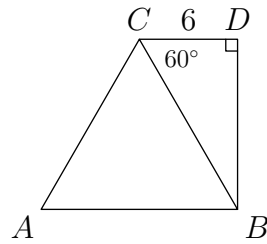
- (a) How many multiples of 4 are between 103 and 217?
- (b) A shopkeeper sells house numbers. She has a large supply of the digits 1, 2, 7, and 8, but no other digits. How many different three-digit house numbers could be made using only the digits in her supply?
- (c) Suppose that we roll two fair 6-sided dice. What is the probability that the two numbers rolled sum to 5?

**Problem Solving**

8. Find the value of
- $x$
- in the diagram below.



9. Rebecca walks 100 feet in a straight line. She then turns 20 degrees to her left and walks another 100 feet, and then turns 20 degrees to her left again. She continues this pattern until she reaches the point where she started. How far did she walk?
10. In rectangle  $ABCD$ , point  $X$  is the midpoint of  $\overline{AD}$  and  $Y$  is the midpoint of  $\overline{CD}$ . What fraction of the area of the rectangle is enclosed by  $\triangle AXY$ ?
11. A 30-60-90 triangle is drawn on the exterior of equilateral triangle  $ABC$  as shown below so that the hypotenuse of the right triangle is one side of the equilateral triangle. If the shorter leg of the right triangle is 6 units, what is  $AD$ ?



12. At Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?
13. At a party, there are only single women and married men with their wives. The probability that a randomly selected woman is single is  $\frac{2}{5}$ . What fraction of the people at the party are married men?

**Don't look at the next page until you've attempted all the problems!**

The answers are below. (The answers to problems in the course will include full detailed solutions as opposed to the mere answers provided below.)

1. (a) 9  
(b) 12  
(c)  $\frac{4}{3}$  or  $1\frac{1}{3}$   
(d)  $2\sqrt{10}$   
(e) 0  
(f)  $23\sqrt{2}$
2. (a)  $60^\circ$   
(b)  $140^\circ$
3. (a)  $\frac{1}{3}$   
(b) 39 square units  
(c)  $72 - 18\pi \text{ in}^2$  (students may approximate this as  $15.45 \text{ in}^2$ )
4. (a)  $\frac{5}{4}$  miles ( $1\frac{1}{4}$  and 1.25 are acceptable)  
(b) 51 feet  
(c) 13
5. (a)  $\angle G = 41^\circ, \angle F = \angle H = 139^\circ$   
(b) 32 inches  
(c) Perimeter: 52 inches, Area:  $120 \text{ in}^2$
6. (a) Average: 90, Median: 91, Mode: 84  
(b) 98
7. (a) 29  
(b) 64  
(c)  $\frac{1}{9}$
8.  $11^\circ$
9. 1800 feet
10.  $\frac{1}{8}$
11.  $6\sqrt{7}$
12. 40
13.  $\frac{3}{8}$