Below is a diagnostic test to determine if students are ready for Art of Problem Solving's **Math 6: Prealgebra (Part 2)** course. We suggest using the following process to assess whether or not your student is ready for the course:

> Step 1: The student should attempt all of the questions below without a calculator and without any help. There is no time limit.
> Step 2: Check the student's responses using the answer key at the end of this document.
> Step 3: The student should be given a second chance on the problems that they answered incorrectly.

A student who is ready for **Math 6: Prealgebra (Part 2)** should be able to answer correctly nearly all of the **Fundamentals** problems and at least half of the **Problem Solving** problems. If a student struggles with these problems, then the student should consider our **Math 6: Prealgebra (Part 1)** course.

Fundamentals

1. **Exponent basics**. Compute:

   (a) 2^5 (b) $(-5)^2 - 4^2$ (c) $3^0 + 3^{-2}$

2. **Exponent laws**. Express each of the following as a power of 2:

   (a) $2^4 \cdot 2^3$ (b) $\dfrac{2^8}{2^4}$ (c) $(2^4)^3$

3. **Number Theory basics**.

   (a) Find the least common multiple of 54 and 24.

   (b) Find the greatest common divisor of 288 and 684.

   (c) Find the least common multiple of 72, 120, and 28.

4. **Fraction arithmetic**. Compute:

   (a) $\dfrac{2}{3} + \dfrac{7}{6} - \dfrac{1}{4}$ (c) $10 \div \dfrac{2}{3}$

   (b) $\dfrac{3}{4} \cdot \dfrac{8}{5} \cdot \dfrac{7}{6}$ (d) $6\dfrac{1}{4} - 3\dfrac{1}{3}$

5. **Fraction and Decimal conversion**. Convert the following decimals to fractions in simplest form:

   (a) 0.625 (b) 3.64 (c) $0.\overline{8}$

   Convert the following fractions to decimals:

   (d) $\frac{13}{4}$ (e) $\frac{7}{20}$ (f) $\frac{5}{6}$

6. **Linear Equations**. Solve each of the following equations:

   (a) $3r - 4 = 16 - 7r$ (b) $\frac{2x-3}{5} = \frac{4-3x}{7}$ (c) $2 - \frac{t}{4} = 3\left(5 - \frac{t}{6}\right)$

7. **Ratio and Rates**.

   (a) The ratio of dogs to cats at an animal shelter is 4 to 5. If the total number of animals (dogs and cats) at the shelter is 108, then how many dogs are at the shelter?

   (b) The ratio of teachers to students in a particular school is 1 to 11. The ratio of 7th grade students to the total number of students is 4 to 9. If there are 396 students in 7th grade, then how many teachers are there?

   (c) A train is traveling 1 mile every 75 seconds. If the train continues at this rate, then how far will it travel in two hours?

Problem Solving

8. What is the value of the sum $5 + 10 + 15 + \cdots + 95 + 100$?
9. Express $2^5 \cdot 8^3 \cdot 16^2$ as a power of 4.
10. The four-digit number $A55B$ is divisible by 36. What is the sum of A and B?
11. Two-fifths of the students at Central Middle School play soccer. One-third of the non-soccer-players play the piano and one-quarter of the soccer-players play the piano. What fraction of the students in Central Middle School play the piano?
12. Douglas writes down his favorite number, which is a two-digit positive integer. He then turns the number into a three-digit number by writing a 7 at the end of his favorite number. This new number is 385 more than Douglas's favorite number. What is Douglas's favorite number?
13. Kayla adds the same number to both the numerator and denominator of the fraction $\frac{1}{10}$. Her resulting fraction equals $\frac{2}{3}$. What number did she add to both the numerator and denominator of her original fraction?
14. Five workers together can build a road in 20 days. Suppose every worker works at the same rate. Three workers work on the road for 10 days before eleven more workers join them. How much longer will it take the fourteen workers to finish the road?

15. A northbound train from Miami to Jacksonville made the 324-mile journey at an average speed of 50 miles per hour. On its southbound return trip, it made the journey at an average speed of 40 miles per hour. To the nearest tenth of a mile per hour, what was the train's average speed for the 648-mile roundtrip journey?

Don't look at the next page until you've attempted all the problems!

The answers to Are You Ready are below. (The answers to problems in the course will include full detailed solutions as opposed to the mere answers provided below.)

1. (a) 32; (b) 9; (c) $1\frac{1}{9}$ or $\frac{10}{9}$
2. (a) 2^7; (b) 2^4; (c) 2^{12}
3. (a) 216; (b) 36; (c) 2520
4. (a) $\frac{19}{12}$ or $1\frac{7}{12}$; (b) $\frac{7}{5}$; (c) 15; (d) $2\frac{11}{12}$ or $\frac{35}{12}$
5. (a) $\frac{5}{8}$; (b) $\frac{91}{25}$ or $3\frac{16}{25}$; (c) $\frac{8}{9}$; (d) 3.25; (e) 0.35; (f) $0.8\overline{3}$
6. (a) $r = 2$; (b) $x = \frac{41}{29}$; (c) $t = 52$
7. (a) 48 dogs; (b) 81 teachers; (c) 96 miles
8. 1050
9. 4^{11}
10. 8
11. $\frac{3}{10}$
12. 42
13. 17
14. 5 days
15. 44.4 miles per hour