

If you can solve nearly all of the following problems with little difficulty *without a calculator*, then the class **Scholars Math 10: Precalculus** would only serve as a review for you.

1. What is the value of $\tan 75^\circ$?
2. Simplify $\cos(12^\circ)\cos(24^\circ)\cos(48^\circ)\cos(96^\circ)$.
3. Let $n \geq 3$ be a positive integer, and let $P_0P_1 \cdots P_{n-1}$ be a regular n -gon inscribed in a circle with radius 1. Compute $P_0P_1 \cdot P_0P_2 \cdot P_0P_3 \cdots P_0P_{n-1}$ in terms of n .
4. What is the value of $\cos^2 10^\circ + \cos^2 50^\circ - \sin 40^\circ \sin 80^\circ$?
5. Find the roots of $z^6 + z^4 + z^2 + 1$.
6. Suppose $\frac{\cos 3x}{\cos x} = \frac{1}{3}$ for some angle x , $0 \leq x \leq \frac{\pi}{2}$. Determine $\frac{\sin 3x}{\sin x}$ for the same x .
7. Find the volume of the tetrahedron with vertices $(-1, 3, 0)$, $(2, 1, 7)$, $(-4, 3, 2)$, $(3, 1, -2)$.
8. A sequence $(a_1, b_1), (a_2, b_2), (a_3, b_3), \dots$ of points in the coordinate plane satisfies

$$(a_{n+1}, b_{n+1}) = (\sqrt{3}a_n - b_n, \sqrt{3}b_n + a_n)$$

for all positive integers n . Suppose that $(a_{100}, b_{100}) = (2, 4)$. What is $a_1 + b_1$?

9. Find $\operatorname{Im}((\cos 12^\circ + i \sin 12^\circ + \cos 48^\circ + i \sin 48^\circ)^6)$.
10. Evaluate $\sin 10^\circ \sin 20^\circ \sin 30^\circ \cdots \sin 170^\circ$.

Don't look at the next page until you've attempted all the problems!

The answers are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

1. $2 + \sqrt{3}$
2. $-\frac{1}{16}$
3. n
4. $\frac{3}{4}$
5. $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, -i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$
6. $\frac{7}{3}$
7. $25/3$
8. $1/2^{98}$
9. 0
10. $\frac{9}{65536}$