

If you've mastered using the Pythagorean Theorem, solving basic equations and systems of linear equations, the basics of logarithms and polynomials, graphing simple functions on the Cartesian coordinate plane, determining angles and arcs in circles, and the arithmetic of complex numbers then you are ready for the Art of Problem Solving Online Class, **Scholars Math 10: Precalculus**. (Answers to these problems are on the following page.)

## Pythagorean Theorem

1. Find the hypotenuse of a right triangle with legs of length 10 and 24.
2. Find the hypotenuse of a right triangle with legs of length 14 and 17.
3. Find the length of the shorter leg of a right triangle with hypotenuse of length 82 and longer leg of length 80.
4. If the hypotenuse of a right triangle has length 60 and one leg has length 45, find the length of the other leg.

## Basics of Complex Numbers

5. Find the value of each of the following
  - (a) The sum of  $3 + 4i$  and  $-9 + 2i$ .
  - (b)  $(4 + 2i)(7 - 6i)$
  - (c)  $(\sqrt{3} + i)^3$
  - (d)  $|3 + 4i|$

## Algebra

6. Find all values of  $x$  that satisfy  $2x^2 - 4x + 1 = 0$ .
7. Solve for  $a$  and  $b$  where  $3a - 2b = -5$  and  $-2a + 3b = 15$ .
8. Find the value of  $10x + 3y$  where  $2x + 5y = 13$  and  $6x - 7y = 11$ .
9. Find all values of  $x$  such that  $\sqrt{x+3} + \frac{3}{\sqrt{x+3}} = 4$ .

## Coordinates and Graphing

10. Point  $A$  is located at  $(3, 5)$  on the Cartesian plane. What is the result of reflecting  $A$  across the  $x$ -axis?
11. Find the points of intersection of the graphs of  $y = 3x^2 + x - 7$  and  $y = 2x^2 + 6x - 13$ .
12. What is the equation of a circle centered at  $(2, 2)$  with radius 5?

13. In a coordinate plane, a circle with center  $(5, 2)$  passes through the point  $(-7, 7)$ . What is the circumference of the circle?

## Sequences and Series

14. Evaluate each of the following.

(a)  $\sum_{k=1}^{10} (6k - 2)$ .

(b)  $\frac{1}{3} \cdot \frac{2}{4} \cdot \frac{3}{5} \cdot \dots \cdot \frac{38}{40}$ .

## Circles

15. What is the measure in degrees of an angle inscribed in a circle that subtends an arc of  $80^\circ$ ?
16. Points  $A$ ,  $B$ , and  $C$  lie on a circle in that order so that the measure of arc  $AB = 110^\circ$  and the measure of arc  $BC = 130^\circ$ . What is the measure in degrees of  $\angle ABC$ ?
17. What is the measure of the angle formed by the minute and hour hands of a clock at 1:30?

## Functions

18. Find  $f(6)$  if  $f(x) = 5 - 2x^2$ .
19. Find the range of  $f(x) = 6 + 2|x|$ .
20. Find the domain of  $f(x) = 3 - \sqrt{2 - x^2}$ .

**Don't look at the next page until you've attempted all the problems!**

The answers are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

- 26
- $\sqrt{485}$
- 18
- $15\sqrt{7}$
- (a)  $-6 + 6i$   
(b)  $40 - 10i$   
(c)  $8i$   
(d) 5
- $1 \pm \frac{\sqrt{2}}{2}$
- $a = 3$  and  $b = 7$
- 37
- $x = -2$  or 6
- (3, -5)
- (2, 7) and (3, 23)
- $(x - 2)^2 + (y - 2)^2 = 25$
- $26\pi$
- (a) 310  
(b)  $\frac{1}{780}$
- $40^\circ$
- $60^\circ$
- $135^\circ$
- 67
- All real numbers greater than or equal to 6. (Alternatively,  $[6, +\infty)$ .)
- All real numbers from  $-\sqrt{2}$  to  $\sqrt{2}$ . (Alternatively,  $[-\sqrt{2}, \sqrt{2}]$ .)