Group theory is the study of symmetry. Objects in nature (physics, chemistry, music, etc.) as well as objects in mathematics itself have beautiful symmetries, and group theory is the algebraic language we use to unlock that beauty. Group theory is the gateway to abstract algebra and tells us (among many other things) that you can't trisect an angle with a straightedge and compass, that there are finitely many perfectly symmetric tiling patterns, and that there is no closed formula for solving a quintic polynomial. In this class we will get a glimpse of the mathematics underlying these famous questions.

This course will focus concretely on building groups from other groups, exploring groups as symmetries of geometric objects, and using the tools of group theory to construct fields. The overarching goal of the course is to learn how modern mathematicians understand a topic as general and seemingly fuzzy as "symmetry".

This course is specifically designed for high-performing students and draws material from many programs for top high school and college students in the country. Our philosophy is that students develop more by learning to solve problems they haven't seen before, as opposed to offering repeated drills that students can memorize their way through. In this way, our classes are structured much more like courses at top-tier colleges.

Textbook(s): Scholars Math 11.1 requires *Groups and Fields* by D. Jeremy Copeland

Sample Problems:

- Find the smallest n for which there exists a permutation f of an n-element set such that f^{1000} is the identity function and 1000 is the least positive integer for which this holds.
- Suppose that G is a group generated by a and b. Suppose that $a^4 = b^7 = aba^{-1}b = e$, $a^2 \neq e$ and $b \neq e$. How many elements of G are of the form c^2 with $c \in G$?
- How many ways are there to color the faces of the octahedron three different colors, up to symmetries of the octahedron?
- Prove that in any regular tiling of the plane, every rotational symmetry is of order 2, 3, 4, or 6.
- Prove that if K is a finite extension field of k, then every element of K is algebraic over k.
- Determine all n for which a regular n-sided polygon can be constructed with ruler and compass.

Time Commitment: 14 lessons, 2 in-class hours + 4–5 hours of homework per lesson.

Grading: 96% weekly Challenge Problems, including Short-Answer and Writing Problems (proofs), and 4% Class Participation.

Content:

Lesson	Scholars Topic
1	Symmetry
2	Examples of Groups
3	Subgroups
4	Abelian Groups
5	Group Actions
6	Orbits and Stabilizers
7	Burnside and Beyond
8	Quotients
9	Functions from Groups to Groups
10	Geometry and Group Theory
11	Field Extensions
12	Polynomials
13	Degree and Constructibility
14	Groups and Fields