Syllabus — Scholars Math 9.2: Intermediate Number Theory

Scholars Math 9.2: Intermediate Number Theory covers number theory using algebraic techniques, multiplicative functions, modular arithmetic, Fermat's/Euler's Theorem, and primitive roots. Much of the first half of the class emphasizes using the basic tools of the Introduction class in clever ways to solve difficult problems. Towards the second half, more theory will be developed, leading students to the beginning olympiad level.

This course is specifically designed for high-performing students and draws material from many programs for top high school students in the country. Our philosophy is that students develop more by learning to solve problems they haven't seen before, as opposed to offering repeated drills that students can memorize their way through. In this way, our classes are structured much more like courses at top-tier colleges.

Textbook(s): Scholars Math 9.2 does not have a required textbook.

Sample Problems:

- A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. If the odometer now reads 002005, how many miles has the car actually traveled?
- Prove that, for all positive integer pairs (a, b) where $b > 2$, $2^b - 1$ does not evenly divide $2^a + 1$.
- Prove that there exists a sequence of 100 different integers such that the sum of the squares of any two consecutive terms is a perfect square.
- Let x and y be relatively prime integers. Show that if p is a prime divisor of $x^2 + xy + y^2$, then p is not $2 \pmod 3$.

Common Core State Standards:

Domain	Subdomain	Standards
Algebra	Seeing Structure in Expressions	4
	Arithmetic with Polynomials & Rational Expressions	4

Time Commitment: 12 lessons, 1.5 in-class hours + 4–5 hours of homework per lesson.

Grading: 64% Short-Answer Challenge Problems, 32% Writing Challenge Problems (proofs), and 4% Class Participation.

Content:

Lesson	Scholars Topic
1	Introduction
2	Bases
3	Divisibility
4	Divisors and Multiplicative Functions
5	Prime Factorizations
6	Algebra in Modular Arithmetic
7	Linear Diophantine Equations
8	Perfect Squares
9	Fermat's Little Theorem and Euler's Theorem
10	Orders and Primitive Roots
11	Quadratic Residues and Squares
12	Sums of Two Squares