AoPS Online
Art of Problem Solving

## Syllabus — Scholars Math 7.2: Introduction to Number Theory

Scholars Math 7.2 is a half-year Introduction to Number Theory curriculum, covering topics like primality, divisibility, the Euclidean Algorithm, number bases, and modular arithmetic. Our philosophy is that students best grow their skills by learning to solve engaging problems, as opposed to offering repeated drills that focus on memorization over critical thinking. In this way, our classes are structured like top-tier college courses.

Note: we currently do not offer Scholars Math 7.2 as a self-paced course.

**Textbook(s):** Scholars Math 7.2 requires *Introduction to Number Theory* by Mathew Crawford.

**Sample Problems:**

- Prove that there exists a sequence of 100 different integers such that the sum of the squares of any two consecutive terms is a perfect square.

- Let $x$ and $y$ be relatively prime integers. Show that if $p$ is a prime divisor of $x^2 + xy + y^2$, then $p$ is not equivalent to $2 \pmod 3$.

**Common Core State Standards:**

The topics in Math 7.2 are not in the Common Core State Content Standards, as most of the material in this course is **beyond** what a typical middle or high school curriculum would cover.

**Time Commitment:** 12 lessons per part, 1.5 hours in-class + 3–5 hours of out-of-class work per lesson.

**Content:**

| 7.2 Lesson | Scholars Topic |
|---|---|
| 1 | Integers, Fractions, Decimals, and Number Bases |
| 2 | Base Number Arithmetic |
| 3 | Multiples, Divisors, and Prime Numbers |
| 4 | Common Factors, Common Multiples, Euclidean Algorithm |
| 5 | Divisor Problems, More with the Euclidean Algorithm |
| 6 | Factorials, Special Integers, Algebra with Integers |
| 7 | Units Digit, Introduction to Modular Arithmetic |
| 8 | Calculations with Modular Arithmetic |
| 9 | Divisibility Rules and Multiplicative Inverses |
| 10 | Multiplicative Inverses, Solving Linear Congruences |
| 11 | Systems of Linear Congruences and the Chinese Remainder Theorem |
| 12 | Number Sense and Applications of Number Theory |